A Discrete Model for Inelastic Deformation of Thin Shells

Yotam Gingold, Adrian Secord, Jefferson Y. Han, Eitan Grinspun and Denis Zorin
Media Research Lab, New York University

**Thin shells**
- Thin shells: thin, curved, flexible surfaces
- Good approximation for real-world objects
- Applications in entertainment, medicine, etc.

**Context**
- Terzopoulos and Fleischer 1988
  - Thin plates using splines
  - Fracture followed mesh edges
- O’Brien and Hodgins 1999, 2002
  - Volumetric meshes for brittle and ductile fracture
- Grinspun, Hirani, Desbrun and Schröder 2003
  - Discrete models using invariants
- Cohen-Steiner and Morvan 2003
  - Discrete curvatures

**Motivation**
- Simulation of thin shells
  - Simple to implement
  - Derivation from fundamental elasticity theory
  - Elegant discretization
  - Captures wide range of materials
- Simple shell model
  - Negligible deformation in the normal direction

**Contributions**
- New discrete bending strain
  - Expressed in terms of mesh invariants
- Applications
  - Elasticity
  - Plastic flow
  - Fracture
- Algorithmic enhancements
  - Search for fracture and collision events
  - Vertex budding
  - Collision response with fracture

**Shape Operator**
- 2nd order tensor — \( \Lambda \)
- For any tangent vector \( v \) on the surface, \( \Lambda v \) is the derivative of the surface unit normal in the direction of \( v \)
- Can be diagonalized for
  - The principal directions of curvature
  - The principal magnitudes of curvature

**Membrane and bending strains**
- Deformations of the shell middle surface
- Integrated over the shell thickness
- Membrane strain: in-plane stretching
- Bending strain: out-of-plane deformation

**Discrete strains**
- Discrete membrane strain
  - Measures change in squared length
  \[
  E_m = \frac{1}{2} \sum (I - r_i^2) (I - T_i) \frac{1}{A} \sum \left( \text{det} \left( \frac{\partial T_i}{\partial s} \right) \right)
  \]
- Discrete bending strain
  - Measures change in curvature magnitude and direction
  \[
  E_b = \frac{1}{2} \sum \left( \text{curl} \left( \frac{\partial T_i}{\partial s} \right) \right) \frac{1}{A} \sum \left( \text{det} \left( \frac{\partial T_i}{\partial s} \right) \right)
  \]

**Budging**
- Fracturing near existing edges can introduce silver triangles
- Vertex budding reparameterizes the mesh
- Move the vertex \( v \) to a location \( v' \) on the fracture line

**Results: Light bulb**
- Glass light bulb anchored into a rigid base
- Struck by a rigid metal ball
- Fracture, collisions and dynamics

**Results: Plasticity**
- Metal tube with plastic absorption of energy
- Realistic permanent denting behavior