Lighting

Physics vs. graphics

Computer graphics terms are somewhat confusing and disagree with physics:

Graphics: color of an object; physics: reflection spectrum (i.e. fraction of light of each frequency that gets reflected).

Graphics: intensity or color of a light ray; physics: radiance distribution (measured in watts/steradian/meter²/meter).

Graphics: intensity or color of a point light source; physics: intensity spectrum of a point light source (almost in agreement!) measured in watts/steradian/meter.
Local lighting model

Describes interaction of the light with the surface.
Almost never truly based on physics: perception plays a greater role.
Visible light: electromagnetic waves, with wavelengths 400nm (violet) - 700nm (red); intensity can vary over many orders of magnitude.
Computer model: only three “frequencies”: RGB, intensity varies over a small range, typically only 255 discrete values/ color.

Flow of light

Assumptions:
light consists out of particles (ignore wave nature)
propagates along straight rays (isotropic medium)

Flow:

\[ N \ v dt \ dA \cos \theta \]

\( N \) particle density
\( dA \) differential area
\( v \) particle velocity
Flux and Flux Density

Flux = particles/unit time; differential flux through a small area:

\[ d\Phi = Nv \cos \theta dA \]

Flux density = particles/(unit time unit area)

\[ \frac{d\Phi}{dA} = Nv \cos \theta \]

Solid Angles

solid angle spanned by a cone is measured by the area of intersection of the cone with a sphere:

\[ \Omega = \frac{A}{R^2} \]

differential solid angle can be assigned a direction. Unit: steradian (full sphere = 4\pi)
Measuring light

For any point in space, we can consider directional distribution of photons going through a differential area at this point.

Radiance: energy per unit time, per unit differential area perpendicular to the ray, per unit solid angle in the direction of the ray.
Measured in watts/meter²/steradian

If \( \phi(x, \omega) = \frac{dN}{d\Omega} \) is directional distribution of photons of wavelength \( \lambda \), going through the area \( \phi(x, \omega) \)
then radiance is \( L(x, \omega, \lambda) = \frac{hc}{\lambda} \phi(x, \omega) \)
energy of a photon

Constancy of Radiance

radiance is constant along a ray: consider the flow of photons in a thin pencil; the number of photons entering on the right with the direction inside \( d\omega \) exit through the other side; equating the expressions for entering and exiting diff. flows we get

\[
d\Phi_1 = L_1 \ d\omega_1 dA_1 = L_2 \ d\omega_2 dA_2 = d\Phi_2 \\
\text{but } dA_1 d\omega_1 = dA_2 d\omega_2 \ \text{so } L_1 = L_2
\]
Wavelength

In natural light all wavelength are present. We cannot represent all of them.

Solution: use perception: light with arbitrary spectrum produces in a perceptual response equivalent to one produced by a linear combination of 3 primary wavelengths (red, green, blue). In most cases, three numbers are ok.

Local illumination model

Describes interaction of the light and the surface

- light can be reflected, absorbed and transmitted
- most important: reflection
- reflection can be ideal specular, diffuse and anything between
- reflection equation relates the outgoing radiance in some direction to the incoming radiance
BRDF

**irradiance:** light flow per unit area of surface

flow of radiance $L$ spanning solid angle $d\omega$ creates differential irradiance $Ld\omega_i \cos \theta_i$

bidirectional reflectance distribution function:

the ratio of reflected radiance in direction $r$ to the differential irradiance in the direction $i$

units: steradians$^{-1}$

$$f(\omega_i, \omega_r) = \frac{dL_r(\omega_i, \omega_r)}{L_i \cos \theta_i d\omega_i}$$

Reflection equation

the outgoing radiance in direction $r$ is the sum of the radiances due to radiance from all incoming directions:

$$L_r(\omega_r) = \int f_r(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i$$

the integral is over the upper hemisphere
Illumination model

Two main components:

- light source characteristics
  - position
  - intensity for each freq. (color)
    often, different intensity can be specified for different colors
  - directional distribution
- surface properties
  - reflectance for each freq. (color)
  - different reflectance can be specified for diffuse and specular light

Reflection geometry

\[ R = 2(N,V)N - V \]
Snell’s law

If a surface separates two media with different refraction indices (e.g. air and water) the light rays change direction when they go through.

**Snell’s law**: the refracted ray is stays in the plane spanned by the normal and the direction of the original ray. The angles between the normal and the rays are related by

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

where \( n_1 \) and \( n_2 \) are refraction indices.

\[
\begin{align*}
\theta_1 & \quad \text{normal} \\
\theta_2 & \\
\end{align*}
\]

Refracted ray direction

\[
T = -nV + \left( n(V,N) - \sqrt{1 - n^2(1 - (V,N)^2)} \right) N
\]

\[
\begin{align*}
V & \\
N & \\
T & \\
\end{align*}
\]
A simple model

In the model commonly used in graphics applications, there are several components:

- **diffuse reflection**: intensity does not depend on the direction to the viewer.

- **specular**: simulates reflective surfaces and specular highlights depending on the direction to the viewer.

- **ambient**: a crude approximation to the illumination created by the light diffusely reflected from surfaces.

Diffuse component

Diffuse surfaces are surfaces following the Lambert’s law: the energy of the light reflected from a surface in a direction $D$ is proportional to the cosine of the angle between the normal and $D$. As intensity (radiance) is proportional to the energy times cross section of the ray, it does not depend on the view direction, but is proportional to the cosine of the angle between the normal and the direction to the light.

$$L_{\text{diff}} = k_{\text{diff}}(L,N)$$

$$A \cos \theta$$

small area $A$
Specular component

Specular component approximates behavior of shiny surfaces. If a surface is an ideal mirror, the light from a source reaches the eye bouncing off a fixed point of the surface, only if the direction to the light coincides with the reflected direction to the eye:

\[
N \quad V \quad L=R
\]

ideal mirror

sphere

Specular reflection

For non-ideal reflectors, the reflection of the light is still the brightest when \( L=R \) but decays, rather than disappears, as the angle between \( L \) and \( R \) increases. One way to achieve this effect is to use cosine of the angle to scale the reflected intensity:

\[
L_{\text{spec}} = k_{\text{spec}} (R,L)^p
\]

Phong exponent
Specular reflection

Natural look of metallic surfaces is difficult to simulate, but the first approximation is obtained using proper highlight color.

For plastic objects, highlights are close in color to the color of the light. For metals, to the color of the surface. Assuming white lights, for plastic set $k_{spec} = [c,c,c]$, where $c$ is a constant, for more metallic look set $k_{spec} = k_{diff}$.
Ambient component

Not all light illuminating a surface comes from light sources, or reflections of light sources in ideal mirrors; however, the light diffusely reflected from other surfaces is difficult to take into account, especially for real-time rendering. It is approximated by the ambient component: a constant is added to all objects. To have more control over ambient contribution, surfaces can be assigned ambient reflectivity.

\[ L_{\text{amb}} = k_{\text{amb}} I_{\text{amb}} \]

Complete equation

\[ I_{\text{total}} = k_{\text{amb}} I_{\text{amb}} + \sum_{\text{all lights}} I_i \left( k_{\text{diff}} \left( L_i, N \right) + k_{\text{spec}} \left( L_i, R \right)^p \right) \]

If we are ray tracing for rendering, in summation only visible lights are present, and there are two additional terms: contribution from the reflected ray and transmitted ray. If we are using Z-buffering, then all active light sources are regarded as visible (OpenGL model)
Attenuation

In real life, radiance reaching us from a light source decreases with distance as $1/r^2$ (the stars are much less bright than the sun). However, due to the nature of approximations used in graphics, the inverse-square law typically results in pictures that are too dark; the fix is to allow the programmer to control how fast the decay is. $I_i$ in the formulas is replaced not by $I_i/r^2$, as physics suggest, but by

$$I_i = \frac{a + br + br^2}{a + br^2}$$

and the most common choice of constants is $a = 1, b = c = 0$, that is, no attenuation!

OpenGL model

Phong exponent called shininess. Several additions:

- emission;
- ambient, diffuse, specular light “intensities”

Setting material parameters $(K_{diff}, K_{spec}, K_{amb}, P)$

Vec3f mat_diffuse, mat_spec, mat_amb;
GLfloat shininess;

...  
glMaterialfv(GL_FRONT, GL_DIFFUSE, mat_diffuse);
glMaterialfv(GL_FRONT, GL_SPECULAR, mat_spec);
glMaterialfv(GL_FRONT, GL_AMBIENT, mat_amb);
glMaterialf(GL_FRONT, GL_SHININESS, shininess);