G22.3033-002: Topics in Computer Graphics: Geometric Modeling Homework #1

New York University March 10, 2004

Homework #1: Splines and Subdivision
Due Date: March 24, 2004

1. Derive expressions for second derivatives of a cubic Bezier curve at the endpoints in terms of its control points.
3. Consider the non-uniform cubic B-spline corresponding to the knot sequence $0, 0, 1, 2, \ldots k - 1, k, k$; this is a curve $f(t)$ defined on $[1, k-1]$. Using blossoming, compute the positions of endpoints $f(1)$ and $f(k-1)$ and tangents at these points.
3. Derive subdivision rules for uniform B-splines of degree four using blossoming, i.e. find expressions for the control points corresponding to the knot sequence $\ldots 0, 1/2, 1, 3/2, 2, \ldots$ in terms of control points corresponding to the sequence
4. Consider a curve subdivision scheme given by the rules identical to cubic B-splines, but with coefficients $1/8, 3/4, 1/8$ replaced with $a, 1-2a, a$, where a is a parameter. a. Write the 5 by 5 subdivision matrix for this scheme. b. Determine for which values of a the scheme converges (the largest eigenvalue magnitude is 1). c. Find the mask for the limit position of a vertex, the left eigenvector of the eigenvalue 1. d. Find the mask for the tangent at a vertex, given by the left eigenvalue of largest eigenvalue less than 1; for which values of a there is more than one such eigenvalue?
5. Suppose a surface is defined using tensor-product basis functions $B(u,v) = B^3(u)B^2(v)$, where B^n is the B-spline basis function of degree n:

$$f(u,v) = \sum_{i,j} p_{ij} B(u-i, v-j)$$

Derive refinement relations for the basis functions and subdivision rules for surfaces.