Physics of light

Local Illumination model

interaction of light with the surface

Need to know

- how to measure light
- how to describe surface properties
- computer representation
Properties of light

- spectrum (energy per wavelength)
- polarization
- coherence

Radiometry: physical properties
Photometry: perceptual properties
Visible wavelengths: 380 nm - 770 nm

Basic Units

Force:
Newton = kg \cdot m/sec^2

Energy:
joule = Newton \cdot m

Power:
watt = joule/sec

To get “standard” eye response, integrate spectrum (energy as function of wavelength) multiplied by relative efficiency.

Luminous energy: talbot;
Luminous power: lumen = talbot/sec
Photometry and Radiometry

Radiometry units are primary. If the spectrum of light $P(\lambda)$ (measured in watts/nm) is known, then luminous power is computed as

$$684 \int V(\lambda)P(\lambda)d\lambda$$

684 is an arbitrary constant measured in lumens/watt (luminosity at the wavelength 555 nm, yellow-green). If most of the energy of a light source is near 555nm, then to convert from watts to lumens multiply by 684.

Flow of light

Assumptions:
- light consists out of particles (ignore wave nature)
- propagates along straight rays (isotropic medium)

Flow:

$$N \, v \, dt \, dA \, \cos \theta$$

- $N$: particle density
- $dA$: differential area
- $v$: particle velocity
Flux and Flux Density

Flux = particles/unit time; differential flux through a small area:

\[ d\Phi = Nv \cos \theta dA \]

Flux density = particles/(unit time unit area)

\[ \frac{d\Phi}{dA} = Nv \cos \theta \]

Solid Angles

Solid angle spanned by a cone is measured by the area of intersection of the cone with a sphere:

\[ \Omega = \frac{A}{R^2} \]

differential solid angle can be assigned a direction. Unit: steradian (full sphere = 4π)
Measuring light

For any point in space, we can consider directional distribution of photons going through a differential area at this point.

Radiance: energy per unit time, per unit differential area perpendicular to the ray, per unit solid angle in the direction of the ray.

Measured in watts/meter²/steradian

If \( \phi(x, \omega) = \frac{dN}{d\omega} \) is directional distribution of photons of wavelength \( \lambda \), going through the area

then radiance is \( L(x, \omega, \lambda) = \frac{hc}{\lambda} \phi(x, \omega) \)

energy of a photon

Constancy of Radiance

Radiance is constant along a ray: consider the flow of photons in a thin pencil; the number of photons entering on the right with the direction inside \( d\omega \) exit through the other side; equating the expressions for entering and exiting diff. flows we get

\[
d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2
\]

but \( dA_1 d\omega_1 = dA_2 d\omega_2 \) so \( L_1 = L_2 \)
**BRDF**

**irradiance:** light flow per unit area of surface

flow of radiance $L$ spanning solid angle $d\omega_i$ creates
differential irradiance $Ld\omega_i \cos \theta_i$

bidirectional reflectance distribution function:

the ratio of reflected radiance in direction $r$ to the
differential irradiance in the direction $i$

units: steradians$^{-1}$

$$f(\omega_i, \omega_r) = \frac{dL_r(\omega_i, \omega_r)}{L_i \cos \theta_i d\omega_i}$$

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**Reflection equation**

the outgoing radiance in direction $r$ is the sum of
the radiances due to radiance from all incoming
directions:

$$L_r(\omega_r) = \int f_r(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i$$

the integral is over the upper hemisphere
Reflection geometry

\[ V = \omega_r, \; L = \omega_i \]
\[ R = L + 2(L \cdot N)N \]

Phong model

1.“BRDF”

\[ f_r(\omega_i, \omega_r) = K_{diff} + K_{spec}(\omega_i \cdot \omega_r)^p \]

Point light source intensity: power per unit solid angle

intensity in a direction \( \omega \):

\[ I(\omega) = \frac{d\Phi}{d\omega} \]

radiance created by light source at distance \( r \) in the direction of the source:

\[ L(\omega, r) = \frac{I(\omega)}{r^2} \]

To avoid integration in the reflection equation, ignore radiance from all directions except a finite number (e.g. direction to the light sources).
Phong model

Phong model and Z-buffer rendering:
- assume point light sources; ignore irradiance from all directions except the directions to the lights;
- ignore occlusions, that is, no shadows).

Phong model and (classical) ray tracing:
- consider reflection and transmission;
- take occlusions into account.

\[ L(\mathbf{V}) = K_{amb} L_{amb} + \sum_i L_i \left( K_{diff} (\mathbf{L}_i \cdot \mathbf{N}) + K_{spec} (\mathbf{R}_i \cdot \mathbf{N}) \right) \]

summation is over all light sources.

Ambient term: a hack. Because we ignore diffuse reflected light from objects (e.g. walls) the resulting images are often too dark.

Another hack: replace \( L_i = \frac{I_i}{r^2} \) with \( \frac{I_i}{d_c + d_r r + d_q r^2} \)
Phong model

Directional plots of BRDF for a fixed incoming direction for different $(K_{diff}, K_{spec}, p)$

- $(0.8,0.2,4)$
- $(0.7,0.3,8)$
- $(0.6,0.4,16)$
- $(0.4,0.6,32)$
- $(0.3,0.7,64)$
- $(0.2,0.8,128)$

Constants and units

- $K_{diff}, K_{spec}$: reflection coefficients, 3 color components
- $p$: Phong exponent, nondimensional, same for all colors
- $L, L_{amb}$: watts/meter$^2$/steradian, 3 color components
- $I_i$: light source intensity, 3 color components
OpenGL model

Several additions:
- ambient term per object;
- emission;
- ambient, diffuse, specular light “intensities”

Setting material parameters \( (K_{\text{diff}}, K_{\text{spec}}, K_{\text{amb}}) \)

\[
\text{GLfloat mat_diffuse[3], mat_spec[3], mat_amb[3];}
\]

\[
\text{GLfloat shininess;}
\]

... 

\[
\text{glMaterialfv(GL_FRONT,GL_DIFFUSE,mat_diffuse);}
\]

\[
\text{glMaterialfv(GL_FRONT,GL_SPECULAR,mat_spec);}
\]

\[
\text{glMaterialfv(GL_FRONT, GL_AMBIENT,mat_amb);}
\]

Lighting model for ray tracing

New effects: reflection, refraction; need more terms

reflection part:

\[
L_1(V) = \sum_{\text{visible sources in front}} L_i (K_{\text{diff}} (L_i \cdot N) + K_{\text{spec}} (R_i \cdot V^p)) + k_{\text{refl}} L_{\text{refl}}
\]

radiance from the reflected ray

\[
L_2(V) = \sum_{\text{visible sources behind}} L_i (K_{\text{diff}} (L_i \cdot N) + K_{\text{spec}} (T_i \cdot V^p)) + k_{\text{trans}} L_{\text{trans}}
\]

radiance from the refracted ray

\[
L(V) = K_{\text{amb}} L_{\text{amb}} + (1 - t)L_1(V) + tL_2(V)
\]

t is transparency