Image shifts

Similar to image stretching:
Interpolate, then sample at new locations.

\[ p_{\text{shifted}}[n] = \sum_{i} h(n + t - i)p[i] \]

Implementation summary

To implement resizing (or shifts)
1. create a temporary image. If resizing by factor \( a \) in \( x \) direction and \( b \) in \( y \) direction, the temporary image should be \( a \times w \) by \( b \times h \), if the original was \( w \) by \( h \). For shifts, use the same size.
2. resize/shift in \( X \) direction using formulas from lectures, computing pixels in the temporary image using pixels of the original image.
3. Create a final image of size \( a \times w \) by \( b \times h \), if resizing, \( w \) by \( h \) if shifting. Resize/shift in \( Y \) direction, computing the pixels of the final image using the pixels of the temporary image.

Image blurring

To blur an image, we do local averaging.
Simplest case:

\[ p_{\text{filtered}}[n] = \sum_{i} h(n - i)p[i] \]

Note: we need values of \( h(t) \) only at integers.

Discrete filtering

When we do not do resampling at arbitrary locations, as we do when resizing the image, we can use discrete filters \( h[i] \), which are just sequences of numbers. One way to obtain such filters is to sample a continuous filter.

Aside from blurring, other effects can be achieved using discrete filters: e.g. edge detection and sharpening.
Discrete filtering

Convolution: Given two discrete sequences \( p[i] \) and \( h[i] \), the convolution of these sequences is a new sequence \( q[i] \) defined by

\[
q[i] = \sum_{j=-\infty}^{\infty} h[j] \cdot p[i-j]
\]

Discrete filtering is convolution of a signal with a filter. (Of course, the summation is not really infinite as both sequences are finite; they are assumed to be extended to both sides by zeros when necessary).

2D convolution

2D convolution:

\[
q[m,n] = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h[i,j] \cdot p[m-i,n-j]
\]

Note: not for any \( h[i,j] \) we can implement 2D convolution as a sequence of 2 1D convolutions. Convolution is implemented as 4 nested loops. Typically, in formulas the indices of filters run in both directions from \(-L\) to \(L\), where \(L\) is an integer. Be careful when retrieving the filter values from an array: you have to convert the range \([-L..L]\) to \([0..2L]\).

Edge detection

Idea: an edge is a sharp change in the image. To find edges means to mark with, say, 255, all pixels which are on an edge. For a continuous image, the places where the intensity changes rapidly, the magnitude of the derivative in some direction is large.

Directional derivatives can be approximated by differences:

\[
\frac{df(x,y)}{dx} = \frac{f(x+1,y) - f(x,y)}{1}
\]

Differences can be computed using filtering.

Difference filters

To compute

\[
\Delta_x[p[n,m]] = p[n+1,m] - p[n,m]
\]

convolve with filter \(h[0,0] = -1, h[-1,0] = 1\), \(h[i,j] = 0\) otherwise.

To compute

\[
\Delta_y[p[n,m]] = p[n,m+1] - p[n,m]
\]

convolve with filter \(h[0,0] = -1, h[0,-1] = 1\), \(h[i,j] = 0\) otherwise.

Edge detection example

To mark locations where the differences are large, compute differences in two directions, square, and threshold:

\[
\text{if } \sqrt{\Delta_x[p[n,m]]^2 + \Delta_y[p[n,m]]^2} > \text{threshold value}
\]

set \(p[n,m]\) to 255, otherwise, to zero.

Note: two intermediate images have positive (white) and negative (black) values.
Sharpening

Idea: to sharpen an image, that is, to make edges more apparent, need to increase the high frequency component and decrease low frequency component. Can be achieved by subtracting a scaled blurred version of the image from the original.

The operation can be done using a single 2D convolution by a filter like this:

\[
\begin{bmatrix}
-1 & -2 & -1 \\
-2 & 19 & -2 \\
-1 & -2 & -1
\end{bmatrix}
\]